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Updating Structural Dynamic Models with Emphasis on the Damping Properties

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The control of complex structural models is a field of growing interest. The problem studied herein concerns damping updating using experimental frequency response functions. Although there are several methods improving structural dynamic models, only a few of them deal specifically with damping improvements. The method introduced is built on mechanical bases, using an error measure on the constitutive relations. The tuning strategy uses an iterative process, each iteration consisting of two steps. The first one is the localization of the erroneous regions. The second step is the correction of the parameters belonging to these regions. Examples illustrate the sensitivity of the error to the damping defects and its effectiveness for updating damped structures.

Nomenclature

$\mathbf{a}$ = damping tensor
$\Re()$ = real part
ε = strain tensor
σ = stress tensor
$\{\}$ = column (dimension n)

Subscripts

a = magnitude
ad = admissible quantity
c = kinematical quantity
d = imposed quantity
E = substructure E
s = statical quantity
sym = symmetric
ω = frequency-domain quantity

Superscripts

$*$ = conjugate and transpose
$(\tilde{\ })$ = experimental quantity

Introduction

NOWADAYS, to predict the dynamic behavior of structures, models are becoming more accurate and more sophisticated. Consequently, validation of these models using experimental information remains a crucial step. Therefore, methods for updating structural dynamic models are undergoing increased attention and intensive development.

Two main categories of methods can be distinguished. The first one consists of the direct methods, correcting the mass and stiffness matrices without really taking into account the geometrical and mechanical characteristics of the structure.[1–5]

The second category includes the parametric methods (or indirect methods), in which the models are corrected by acting on the structural parameters. Generally, these methods are based either on the input equation such as in Refs. 6 and 7 or on the output equation such as in Refs. 8 and 9. These methods are commonly called *sensitivity methods* because they make use of the sensitivity terms of the eigenfrequencies, of the eigenmodes, or of cost functions, with respect to the design parameters.

There is another family of methods wherein the approach we are developing can be positioned. It is an extension to the works conducted in the field of a posteriori error estimators in an effort to quantify the quality of a finite element computation.[10] The method we are developing is based on the concept of the error on the constitutive relations. A prior study focused on its ability to deal with the free vibration case.[11]

Recently, this method has been theoretically extended to a wide range of problems.[12,13] It can incorporate either the damping or the nonlinearities due to both materials and contact. Moreover, it is able to assimilate different types of experimental information (responses to static loads or modal and forced vibrations). The major characteristic of our method, with respect to those cited previously, is that the experimental measurements are not the only reference in quantifying the dynamic model's quality. The reliable equations are distinguished from the less reliable ones; for example, the equilibrium equation is assumed to be reliable, whereas the constitutive relations are not assumed to be so reliable. Therefore, the equilibrium equation enters into what is called the reference. Concerning the experimental data, the more accurate information is also to be distinguished from the less accurate information; for example, the frequencies and the sensor's locations are assumed to be more accurate data. Hence, to quantify the quality of the model with respect to the experimental data and the mechanical principles, a modified error on the constitutive relations is defined, whereby the reliable equations and the more accurate data are verified exactly. As a result, the most erroneous regions are those providing the most significant contribution to the error.

The updating process is iterative. Each iteration consists of two steps. The first one is the localization of the most erroneous regions. This step is performed using local indicators built on the error on the constitutive relations, which are very different from the classical sensitivity indicators of optimization tools. The second step involves the correction of the few parameters belonging to these regions.

This paper is devoted to damping updating. At present, this structural characteristic is the least understood aspect within the updating field. In contrast to mass and stiffness properties, damping adheres to laws that are still not well known. The determination of damping properties using experiments is, thus, unavoidable. Over the past several years, damping updating has been studied by many authors, such as in Refs. 14 and 15, where the identified complex modes are used, and in Refs. 9 and 16, which utilize frequency response functions.

After recalling the concept of the error on the constitutive relations, examples concentrating on lightly damped structures are presented. The mass and stiffness matrices are assumed to have been updated in a preliminary stage and are satisfactorily known. Hence, these examples display damping. These models are linear, and the experimental data are noisy, incomplete frequency response functions. The examples serve to illustrate the capabilities and limits of

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the updating process. The influence of residuals defaults in the mass and stiffness properties on the damping correction is also studied.

The method is being deliberately introduced on the continuous problem so as to highlight the foundations of the error on the constitutive relations.

Error on the Constitutive Relations

Before introducing the error on the constitutive relations, we will describe, for a studied structure, the dynamic problem for small perturbations.

Dynamic Problem to Be Solved

Let us consider a structure described by a domain Ω during the time interval $[0, T]$. On the boundary $\partial\Omega$ of the structure, the displacements $\boldsymbol{U}_d$ and the forces $\boldsymbol{F}_d$ are described on $\partial_1\Omega$ and $\partial_2\Omega$, respectively. Body forces $\boldsymbol{f}_d$ are given in Ω. The problem to solve during $[0, T]$ can then be written as follows.

Find the displacement $\boldsymbol{U}(\boldsymbol{M}, t)$, the stress $\boldsymbol{\sigma}(\boldsymbol{M}, t)$, and the density $\boldsymbol{\Gamma}(\boldsymbol{M}, t)$, $t \in [0, T]$, $\boldsymbol{M} \in \Omega$ such that they satisfy the following:

The kinematic constraints and the initial conditions

$$\forall t \in [0, T], \qquad \boldsymbol{U}|_{\partial_1\Omega} = \boldsymbol{U}_d \tag{1a}$$

$$\forall \boldsymbol{M} \in \Omega, \qquad \boldsymbol{U}|_{t=0} = \boldsymbol{U}_0 \tag{1b}$$

$$\left.\frac{\mathrm{d}\boldsymbol{U}}{\mathrm{d}t}\right|_{t=0} = \dot{\boldsymbol{U}}_0 \tag{1c}$$

$$\boldsymbol{U} \in \mathcal{U}^{[0,T]} \tag{1d}$$

The equilibrium equation

$$\forall t \in]0, T[, \qquad \forall \boldsymbol{U}^* \in \mathcal{U}_0$$

$$\int_\Omega \boldsymbol{\Gamma} \cdot \boldsymbol{U}^* \,\mathrm{d}\Omega = -\int_\Omega \operatorname{tr} \boldsymbol{\sigma}\varepsilon(\boldsymbol{U}^*)\,\mathrm{d}\Omega + \int_\Omega \boldsymbol{f}_d \cdot \boldsymbol{U}^* \,\mathrm{d}\Omega + \int_{\partial_2\Omega} \boldsymbol{F}_d \cdot \boldsymbol{U}^* \,\mathrm{d}S \tag{2a}$$

$$\boldsymbol{\sigma} \in \mathcal{S}^{[0,T]}, \qquad \boldsymbol{\Gamma} \in \Gamma^{[0,T]} \tag{2b}$$

The constitutive relations

$$\forall t \in]0, T[, \qquad \forall \boldsymbol{M} \in \Omega$$

$$\boldsymbol{\sigma}|_t = \mathcal{A}(\dot{\varepsilon}(\boldsymbol{U})|_\tau;\, \tau \le t) \tag{3a}$$

$$\boldsymbol{\Gamma}|_t = \rho\frac{\mathrm{d}^2 U}{\mathrm{d}t^2} + \boldsymbol{a}(\dot{\boldsymbol{U}}|_\tau;\, \tau \le t) \tag{3b}$$

where $\mathcal{U}^{[0,T]}$, $\mathcal{S}^{[0,T]}$, and $\Gamma^{[0,T]}$ are the spaces where the displacement, the stress, and the density force are defined, respectively,

$$\varepsilon(\boldsymbol{U}) = [\operatorname{grad} \boldsymbol{U}]_{\mathrm{sym}}, \qquad \mathcal{U}_0 = \{\boldsymbol{U} \text{ null on } \partial_1\Omega, \boldsymbol{U} \in \mathcal{U}\}$$

and ρ is the density, assumed here to be constant with respect to time. Further on, the constitutive relations are assumed to satisfy the Drucker stability conditions.[17] These conditions ensure the uniqueness aspect of the problem, and they are satisfied by a large class of materials.

Error on the Constitutive Relations

The notion of the error on the constitutive relations is based on a rearrangement of the problem equations into two groups: 1) the kinematic constraints and the equilibrium equation, assumed to be the reliable equations; and 2) the constitutive relations, assumed to be the less reliable equations.

To define the error on the constitutive relations, we introduce the triplet s, displacements–stress–force, in $\Omega \times [0, T]$. A triplet $s(\boldsymbol{U}, \boldsymbol{\sigma}, \boldsymbol{\Gamma})$, defined in $\Omega \times [0, T]$, is admissible if it satisfies the reliable equations, meaning the kinematic constraints and initial conditions, along with the equilibrium equations. The corresponding space is then denoted $\mathcal{S}_{\mathrm{ad}}^{[0,T]}$. Hence, the problem can be written as follows.

Find $s \in \mathcal{S}_{\mathrm{ad}}^{[0,T]}$ satisfying the constitutive relations

$$\boldsymbol{\sigma}|_t = \mathcal{A}(\dot{\varepsilon}(\boldsymbol{U})|_\tau;\, \tau \le t), \qquad \boldsymbol{\Gamma}|_t = \rho\frac{\mathrm{d}^2\boldsymbol{U}}{\mathrm{d}t^2} + \boldsymbol{a}(\dot{\boldsymbol{U}}|_\tau;\, \tau \le t) \tag{4}$$

Such a triplet involves two kinds of quantities: statical quantities $(\boldsymbol{\Gamma}, \boldsymbol{\sigma})$, denoted $(\boldsymbol{\Gamma}_s, \boldsymbol{\sigma}_s)$, and a kinematical quantity $\boldsymbol{U}$, denoted $\boldsymbol{U}_c$.

Using the constitutive relations and the initial conditions, the admissible triplet s can be related to $(\boldsymbol{\sigma}_c, \boldsymbol{\Gamma}_c)$ and $(\varepsilon_s, \boldsymbol{U}_s)$ such that

$$\boldsymbol{\sigma}_s|_t = \mathcal{A}(\dot{\varepsilon}(\boldsymbol{U}_s)|_\tau;\, \tau \le t), \qquad \boldsymbol{\sigma}_c|_t = \mathcal{A}(\dot{\varepsilon}(\boldsymbol{U}_c)|_\tau;\, \tau \le t)$$

$$\boldsymbol{\Gamma}_s|_t = \rho\frac{\mathrm{d}^2\boldsymbol{U}_s}{\mathrm{d}t^2} + \boldsymbol{a}(\dot{\boldsymbol{U}}_s|_\tau;\, \tau \le t) \tag{5}$$

$$\boldsymbol{\Gamma}_c|_t = \rho\frac{\mathrm{d}^2\boldsymbol{U}_c}{\mathrm{d}t^2} + \boldsymbol{a}(\dot{\boldsymbol{U}}_c|_\tau;\, \tau \le t)$$

We define at time t

$$\eta_t^2(s) = \sup_{\tau \le t} \int_0^\tau \int_\Omega \{(1-\gamma)(\boldsymbol{\Gamma}_c - \boldsymbol{\Gamma}_s)\cdot(\dot{\boldsymbol{U}}_c - \dot{\boldsymbol{U}}_s) + \gamma \operatorname{tr}[(\boldsymbol{\sigma}_c - \boldsymbol{\sigma}_s)(\varepsilon(\dot{\boldsymbol{U}}_c) - \dot{\varepsilon}_s)]\}\,\mathrm{d}t\,\mathrm{d}\Omega \tag{6}$$

If the constitutive relations satisfy the Drucker stability conditions, $\eta_t(s)$ is called the global error on the constitutive relations. Then it is demonstrated in Ref. 12 that the following propositions are equivalent: 1) $s = (\boldsymbol{U}, \boldsymbol{\sigma}, \boldsymbol{\Gamma})$ is the exact solution to the problem over $[0, T]$ and 2) $\eta_t(s) = 0$ for $t \in [0, T]$; $s \in \mathcal{S}_{\mathrm{ad}}^{[0,T]}$.

Hence, the problem can be rewritten as follows:

$$\begin{gathered}\text{find } s \in \mathcal{S}_{\mathrm{ad}}^{[0,T]} \\ \text{minimizing } \eta_t^2(s') \quad \text{with} \quad s' \in \mathcal{S}_{\mathrm{ad}}^{[0,T]}\end{gathered} \tag{7}$$

note that γ is a parameter belonging to $[0, 1]$ that depends on the reliability of the relations (3a) and (3b) of the analytical model. Its current value is 0.5.

Application to Model Updating

To take account of results generated from either static or vibration tests, the data considered in the problem are assumed to be harmonic:

$$\Re\big(\boldsymbol{U}_{d\omega}\, e^{i\omega t}\big), \qquad \Re\big(\boldsymbol{f}_{d\omega}\, e^{i\omega t}\big), \qquad \Re\big(\boldsymbol{F}_{d\omega}\, e^{i\omega t}\big)$$

where ω is a given angular frequency. The magnitudes ascribed to $\mathbf{C}$ (complex numbers) are the unknowns of the problem. For the sake of simplicity, we will employ the same notation later for the associated amplitudes and the quantities. The space of the admissible amplitudes is denoted $\mathcal{S}_{\mathrm{ad},\omega}$. Here, $*$ designates the conjugate quantities. Writing the relations (3a) and (3b) as

$$\boldsymbol{\sigma} = K\varepsilon + B\dot{\varepsilon}, \qquad \boldsymbol{\Gamma} = \rho\frac{\mathrm{d}^2\boldsymbol{U}}{\mathrm{d}t^2} + \mathbf{a}\dot{\boldsymbol{U}} \tag{8}$$

enables us to treat classical linear damping. K is the Hooke operator; $\mathbf{a}$ and B are real, linear, symmetric, and positive definite operators that serve to satisfy the Drucker stability conditions. Hence, damping can be proportional, nonproportional, viscous, or hysteretic.

To rewrite the error on the constitutive relations in the frequency domain, as well as to take into account the relations in Eq. (8), let us first consider the term

$$E_1 = \int_0^\tau \int_\Omega (\boldsymbol{\Gamma}_c - \boldsymbol{\Gamma}_s)\cdot(\dot{\boldsymbol{U}}_c - \dot{\boldsymbol{U}}_s)\,\mathrm{d}\Omega\,\mathrm{d}t \tag{9}$$

where

$$\boldsymbol{\Gamma}_c - \boldsymbol{\Gamma}_s = \rho\frac{\mathrm{d}^2(\boldsymbol{U}_c - \boldsymbol{U}_s)}{\mathrm{d}t^2} + \mathbf{a}(\dot{\boldsymbol{U}}_c - \dot{\boldsymbol{U}}_s) \tag{10}$$

Then E_1 becomes

$$E_1 = \frac{\rho}{2}\int_\Omega (\dot{\boldsymbol{U}}_c - \dot{\boldsymbol{U}}_s)^2\big|_\tau\,\mathrm{d}\Omega + \int_0^\tau\int_\Omega \mathbf{a}(\dot{\boldsymbol{U}}_c - \dot{\boldsymbol{U}}_s)^2\,\mathrm{d}\Omega\,\mathrm{d}t \tag{11}$$

If

$$U_c = \tfrac{1}{2}\left(U_{ac}\, e^{i\omega t} + U_{ac}^*\, e^{-i\omega t}\right) \tag{12}$$

$$U_s = \tfrac{1}{2}\left(U_{as}\, e^{i\omega t} + U_{as}^*\, e^{-i\omega t}\right) \tag{13}$$

then

$$\dot{U}_c - \dot{U}_s = (i\omega/2)\left[(U_{ac} - U_{as})\, e^{i\omega t} - (U_{ac} - U_{as})^* e^{-i\omega t}\right] \tag{14}$$

and

$$\int_0^\tau \int_\Omega \mathbf{a}(\dot{U}_c - \dot{U}_s)^2 \,\mathrm{d}\Omega\,\mathrm{d}t$$

$$= \frac{\tau}{2}\omega^2 \int_\Omega \mathbf{a}(U_{ac} - U_{as})^*(U_{ac} - U_{as})\,\mathrm{d}\Omega \tag{15}$$

Hence,

$$E_1 = \int_\Omega \left\{ \frac{\rho}{2}\omega^2 (U_{ac} - U_{as})^*(U_{ac} - U_{as}) + \frac{\tau}{2}\omega^2 \mathbf{a}(U_{ac} - U_{as})^*(U_{ac} - U_{as}) \right\} \mathrm{d}\Omega \tag{16}$$

Now, if we consider

$$E_2 = \int_0^\tau \int_\Omega \mathrm{tr}[(\boldsymbol{\sigma}_c - \boldsymbol{\sigma}_s)(\varepsilon_c - \varepsilon_s)]\,\mathrm{d}\Omega\,\mathrm{d}t \tag{17}$$

we can demonstrate in an analogous way that

$$E_2 = \int_\Omega \frac{1}{2}\,\mathrm{tr}\left[(K + \tau\omega^2 B)(\varepsilon_{ac} - \varepsilon_{as})^*(\varepsilon_{ac} - \varepsilon_{as})\right]\mathrm{d}\Omega \tag{18}$$

where

$$\varepsilon_c = \tfrac{1}{2}\left(\varepsilon_{ac}\, e^{i\omega t} + \varepsilon_{ac}^*\, e^{-i\omega t}\right) \tag{19}$$

$$\varepsilon_s = \tfrac{1}{2}\left(\varepsilon_{as}\, e^{i\omega t} + \varepsilon_{as}^*\, e^{-i\omega t}\right) \tag{20}$$

Finally, the error on the constitutive relations becomes

$$\eta_\omega^2(s) = \frac{1-\gamma}{2}\int_\Omega \omega^2(\rho + T\mathbf{a})(U_c - U_s)^{(*)} \cdot (U_c - U_s)\,\mathrm{d}\Omega + \frac{\gamma}{2}\int_\Omega \mathrm{tr}\left[(K + T\omega^2 B)(\varepsilon(U_c) - \varepsilon_s)^{(*)}(\varepsilon(U_c) - \varepsilon_s)\right]\mathrm{d}\Omega \tag{21}$$

for $s \in \mathcal{S}_{\mathrm{ad}}^{[0,T]}$ and $t \in [0, T]$.

It is shown in Ref. 12 that, if $\eta_\omega(s)$ is null, then s is the solution of the problem. Problem (7) then becomes, for a given frequency ω,

$$\begin{gathered} \text{find } s_\omega \in \mathcal{S}_{\mathrm{ad},\omega} \\ \text{minimizing } \eta_\omega^2(s') \quad \text{with} \quad s' \in \mathcal{S}_{\mathrm{ad},\omega} \end{gathered} \tag{22}$$

Modified Error on the Constitutive Relations

The main problem is to define a quality measure for comparing the model with the experimental information. At this stage, many limitations can be encountered. The first one is the incompleteness of the experimental measurements. Generally, to overcome this problem, the experimental knowledge is enlarged by the exploitation of data generated from either modifications of the experimental conditions[18,19] or the perturbation of the model.[20]

Another limitation is the unavoidable additive measurement noise. Thus, it becomes necessary to distinguish the more accurate information from less accurate information. The experimental results consist of the angular frequency ω, the force $\tilde{F}_{d\omega}$, and the measured displacements $\tilde{U}_\omega$ denoted $\Pi\tilde{U}_\omega$; Π is a projection operator that details the measured part of the displacements. We assume that the force $\tilde{F}_{d\omega}$ is imposed at only one location. This means that its direction and location are assumed to be accurate information. The measured displacements are assumed to be normalized by the magnitude of the force $\tilde{F}_{d\omega}$. The case of a general force vector is treated in Ref. 21.

Regarding these assumptions, a global modified error on the constitutive relations can be defined that enables us to compare the analytical model with the experimental results; for a given frequency ω, this error can be written as follows:

$$e_\omega^2(s) = \eta_\omega^2(s) + (r/1-r)\|\Pi U_c - \Pi\tilde{U}_\omega\|^2 \tag{23}$$

where $\|\,\|^2$ is an energetic norm [see Eqs. (35) and (36)] and $\eta_\omega^2(s)$ contains all of the accurate quantities: the equilibrium equation and the accurate experimental information. The less accurate experimental data are within the second term, influenced by the coefficient $r/(1-r)$, where r is a parameter belonging to [0, 1] whose value depends on the reliability of the experimental information provided in the second term. The current value is $r = 0.5$.

The problem presented in Eq. (7) can then be extended to the following:

$$\begin{gathered} \text{find } s_\omega \in \mathcal{S}_{\mathrm{ad},\omega} \\ \text{minimizing } e_\omega^2(s') \quad \text{with} \quad s' \in \mathcal{S}_{\mathrm{ad},\omega} \end{gathered} \tag{24}$$

Achieving an Updated Model with Respect to a Set of Measurements

Once the error measure (23) has been defined, a representative model for the structure remains to be developed, given that experimental data are indeed available, over a frequency range $[\omega_{\min}, \omega_{\max}]$. The method chosen to incorporate these experimental data will depend essentially on the purpose of the analytical model. In the case of the error on the constitutive relations, a technique that acts in two interactive processes has been employed. The first one is called scaling, and the second one is called weighting.

Scaling Process

The aim of this process is to scale the error on the constitutive relations (23) in such a way as to obtain the same levels of error throughout the frequency range $[\omega_{\min}, \omega_{\max}]$.

To begin, let us recall that the error on the constitutive relations is a function that has an energetic interpretation. Given that the shapes and levels of the peak resonances are affected mainly by the damping properties of the structure, the error on the constitutive relations can be thought of as a dissipation error in the vicinities of the resonance frequencies. Therefore, it seems natural to scale it with respect to the dissipated energy (in one cycle) at these frequencies. The dissipated energy is a quantity that in practice is easy to measure at a given frequency ω. The proposed procedure seeks to identify the forces to be applied to the structure in such a way as to produce the same dissipated energy in the vicinities of the resonances. Then the excitation force magnitude is interpolated linearly between two successive resonance frequencies. We should recall at this point that the structures under investigation are assumed to be linear. Several weighting techniques have already been studied, such as in Ref. 22. A comparison between this technique and the earlier one is made in the first example.

Weighting Process

The error on the constitutive relations now has the same level throughout the experimental bandwidth. It is, thus, possible to update the analytical model by focusing on some selected frequencies, depending on the purpose of the model, without neglecting the other frequencies.

Let us consider a function z depending on the frequency ω, scaled such that

$$\int_{\omega_{\min}}^{\omega_{\max}} z(\omega)\,\mathrm{d}\omega = 1, \qquad z(\omega) \geq 0 \tag{25}$$

Thus, a global error on the constitutive relations can be written as follows:

$$\varepsilon^2 = \int_{\omega_{\min}}^{\omega_{\max}} z(\omega)\eta_\omega^2(s)\,\mathrm{d}\omega \tag{26}$$

where $z(\omega)$ is the weighting factor at the frequency ω. If the structure is divided into substructures $E \in \mathbf{E}$, then the contribution ε_E of the substructure E to the error is defined as follows:

$$\varepsilon_E^2 = \int_{\omega_{\min}}^{\omega_{\max}} z(\omega)\,\eta_{\omega,E}^2(s)\,\mathrm{d}\omega, \qquad \eta_\omega^2(s) = \sum_{E\in\mathbf{E}} \eta_{E,\omega}^2(s) \tag{27}$$

Adaptive Improvement of the Model

The analytical model depends on structural parameters that are not necessarily well defined: Young's modulus, thickness, damping coefficients, etc. The problem now is to find the right values of these parameters from the experimental data. However, this is an ill-posed, inverse problem. This is the reason why an iterative process has been proposed. Each iteration consists of two steps: 1) localization of the erroneous regions and 2) correction of the erroneous regions.

Localization Step

In the initial investigations related to the error on the constitutive relations,[23] the importance of the localization step has been highlighted. At the present time, for several reasons, the methods making use of a localization step are witnessing a growing interest.[24] First, the highly underdetermined nature of the updating task can be overcome because this stage enables us to successively select the most erroneous regions. Moreover, this feature is well suited in health monitoring applications to detect damaged members in the structures.

From the model updating standpoint, there are two common approaches for finding the parameters to be updated.[25] The first seeks the smallest changes necessary in the mass and stiffness matrices. This kind of method generally leads to corrections without a physical significance. The second consists of selecting a set of parameters and then determining the changes in the values necessary to make the model match the test data.

In our case, the choice of the parameters is based on the energetic error indicator ε_E introduced in Eq. (27), where ε_E represents the contribution of the substructure E to the defined error on the constitutive relations. Hence, the regions where the error is the most significant are the more erroneous regions. Because these regions have been localized, the selection of the parameters to update can be performed by computing the sensitivity terms of the error with respect to the structural parameters that belong to these regions.

Correction Step

Once the structural parameters $\boldsymbol{k}$ to be modified have been identified, the mathematical model can be corrected. If $\boldsymbol{k} \in [\mathcal{K}]_z$ (where z are the substructures to correct), the problem is as follows:

$$\begin{gathered} \text{find } \boldsymbol{k} \in [\mathcal{K}]_z \\ \text{minimizing } e^2(\boldsymbol{k}') \quad \text{with} \quad \boldsymbol{k}' \in [\mathcal{K}]_z \end{gathered} \tag{28}$$

where

$$e^2(\boldsymbol{k}) \equiv \sum_{\omega_{\max}}^{\omega_{\min}} z(\omega) e_\omega^2(s) \tag{29}$$

In this study, the Broyden–Fletcher–Goldfarb–Shanno method has been applied to minimize the function error on the constitutive relations.

Finite Element Discretization

Let us associate the displacements fields $(\boldsymbol{U}, \boldsymbol{V}, \boldsymbol{W})$ to the admissible triplet $(\boldsymbol{U}_c, \boldsymbol{\sigma}_s, \boldsymbol{\Gamma}_s)$ such that

$$\begin{gathered} \boldsymbol{U}_c = \boldsymbol{U}, \qquad \boldsymbol{\sigma}_s = K\varepsilon(\boldsymbol{V}) + i\omega B(\boldsymbol{V}) \\ \boldsymbol{\Gamma}_s = -\rho\omega^2 \boldsymbol{W} + i\omega \mathbf{a}(\boldsymbol{W}) \end{gathered} \tag{30}$$

The finite element discretization of problem (24) leads to building of a mass matrix and a stiffness matrix denoted M and K, respectively. Damping matrices A and B related to $\mathbf{a}$ and B are also built and can be frequency dependent. Let $(\{U\}, \{V\}, \{W\})$ be the nodal values of the displacements fields. The error $\eta_\omega^2(s)$ defined in Eq. (21) is then rewritten as follows:

$$\begin{aligned} \eta_\omega^2(s) &= (\gamma/2)(\{U\} - \{V\})^{(*)}(K + T\omega^2 B)(\{U\} - \{V\}) \\ &+ [(1-\gamma)/2]\omega^2(\{U\} - \{W\})^{(*)}(M + TA)(\{U\} - \{W\}) \end{aligned} \tag{31}$$

Here $*$ designates the conjugate transpose. A triplet $(\{U\}, \{V\}, \{W\})$ is admissible if it satisfies

$$(K + i\omega B)\{V\} + (-\omega^2 M + i\omega A)\{W\} = \{F\} \tag{32}$$

where $\{F\}$ is the generalized force vector. We assume that the imposed displacements are restrained to zero.

Until now, $\eta_\omega(s)$ has been discretized using the finite element method. Let us explicitly write the energetic norm $\|\Pi \boldsymbol{U}_c - \Pi \tilde{\boldsymbol{U}}_\omega\|$. The measured displacement amplitudes are $\Pi \tilde{\boldsymbol{U}}_\omega$; the operator Π is a matrix defined as follows:

$$\begin{gathered} \Pi_{ii} = 1 \quad \text{if the dof } i \text{ is measured} \\ \Pi_{ii} = 0 \quad \text{if the dof } i \text{ is not measured} \\ \Pi_{ij} = 0 \quad \text{if} \quad i \neq j \end{gathered} \tag{33}$$

Example: Consider a system with four degrees of freedom (DOFs) in which both the first and the third ones are not measured. Then

$$\Pi\{\tilde{U}\}_\omega = \begin{bmatrix} 0 & & & \\ & 1 & & \\ & & 0 & \\ & & & 1 \end{bmatrix} \begin{Bmatrix} - \\ \tilde{U}_{\omega 2} \\ - \\ \tilde{U}_{\omega 4} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \tilde{U}_{\omega 2} \\ 0 \\ \tilde{U}_{\omega 4} \end{Bmatrix} \tag{34}$$

The norm $\|\Pi \boldsymbol{U}_c - \Pi \tilde{\boldsymbol{U}}_\omega\|$ is then rewritten using the mass, stiffness, and damping matrices as follows:

$$\|\Pi\{U\} - \Pi\{\tilde{U}\}_\omega\|^2 = (\Pi\{U\} - \Pi\{\tilde{U}\}_\omega)^{(*)} G_\omega (\Pi\{U\} - \Pi\{\tilde{U}\}_\omega) \tag{35}$$

where

$$G_\omega = [(1-\gamma')/2]\omega^2(M + TA) + (\gamma'/2)(K + T\omega^2 B) \tag{36}$$

The current value of γ' is 0.5.

Localization Step at Iteration *n*

Next, at an iteration n, the localization step can be written as follows.

Find the substructures $E \in \mathbf{E}$ satifying

$$\varepsilon_E \geq 0.8 \max_{E \in \mathbf{E}} \varepsilon_E \tag{37}$$

Correction Step at Iteration *n*

At an iteration n, the correction step is as follows.

Find

$$\min_{(\{U\},\{V\},\{W\},\{k\})} \sum_{\omega_{\min}}^{\omega_{\max}} z(\omega) e_\omega^2(s, \{k\}) \tag{38}$$

with the constraints

$$(K(\{k\}) + i\omega B(\{k\}))\{V\} + (-\omega^2 M(\{k\}) + i\omega A(\{k\}))\{W\} = \{F\}$$

Here problem (38), related to problem (28), is not linear.

Remark: In the case of proportional damping, it is possible to reduce the size of problems (37) and (38) using a truncated modal basis associated to the conservative system.

Examples

The experimental results used in the following examples have been simulated from the mathematical model in which some of the structural parameters have been modified. Moreover, the function z is assumed to be constant so as not to privilege certain frequencies.

Error Sensitivity to Residual Stiffness and Mass Defects

The aim of this example is to investigate the influence of the scaling process introduced in this paper with the scaling process used in previous studies, such as Ref. 22, on the error sensitivity to residual stiffness and mass defects.

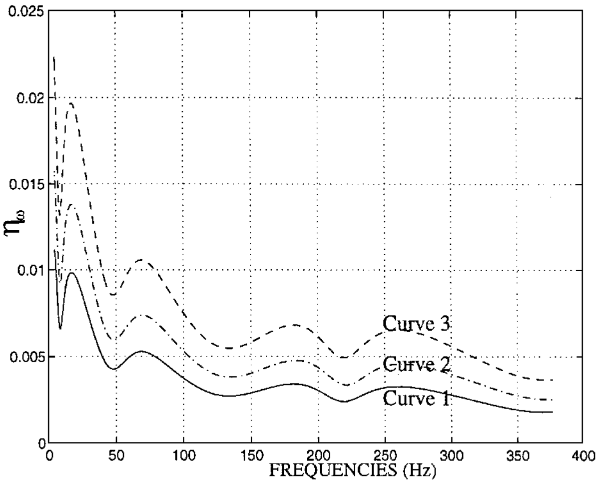
Let us consider a clamped free beam, discretized into 15 two-dimensional beam elements. The characteristics of the structure are given in Table 1. The force location is at the free end of the beam. To compare the scaling processes, we will investigate the sensitivity of the scaled error on the constitutive relations with respect to mismodeled damping parameters associated with residual stiffness errors (which can remain after updating of the mass or stiffness matrices). Then three curves can be plotted for each scaling process:

Table 1 Beam characteristics

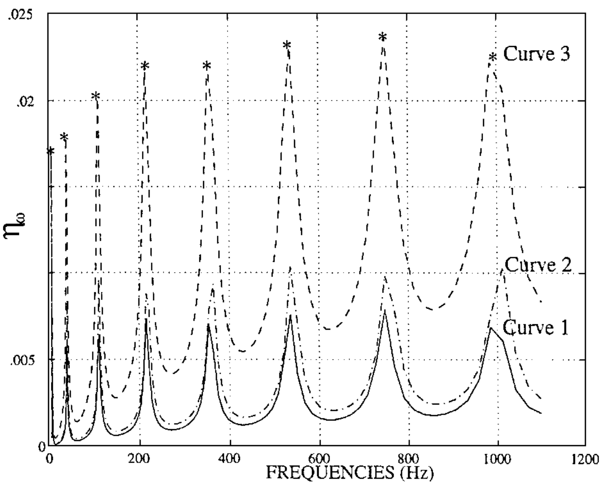
Total length	0.8 m
Section	10^{-4} m^2
Young's modulus	$0.31\,10^8$ N/m^2
Density	7800 kg/m^3
Damping	Hysteretic
Damping loss factor	1%

Table 2 Truss characteristics

Dimensions	$8 \times 1 \times 1$ m^3
Young's modulus	$0.75\,10^7$ N/m^2
Density	2800 kg/m^3
Damping	Hysteretic
Damping loss factor	1%
Frequency bandwidth	[0, 100] Hz



a) Previous error sensitivity to residual stiffness and mass defects



b) New error sensitivity to residual stiffness and mass defects

Fig. 1 Comparison of scaling processes.

1) where the damping loss factor is increased by 100%, 2) where the damping loss factor is increased by 100% and the Young's modulus is increased by 2%, and 3) where the damping loss factor is increased by 200%.

Figure 1b shows that the revised scaling process enables us to obtain the same levels of the error at the different resonance frequencies. Moreover, the error's peaks are located at the resonance frequencies, as could be expected. Finally, it can be remarked (Fig. 1a) that the sensitivity of the previous technique[22] to both the residual stiffness error and the damping error is the same, whereas the sensitivity of the revised scaling process to the damping has been greatly improved (see Fig. 1b).

Updating Eight-Bay Truss Damping Coefficients

This example deals with the damping updating of the structure presented in Fig. 2. The characteristics of the truss are given in Table 2. The structure has been discretized into 109 bar elements. The experimental results have been simulated on the same structure, in which the damping loss factor has been set at 10^{-4}. The measurements are supposed to have been taken from 10 regularly spaced sensors, i.e., less than 10% of the total number of DOFs. A

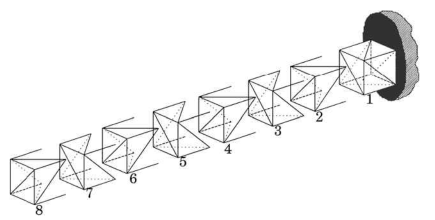


Fig. 2 Eight-bay truss structure and its decomposition.

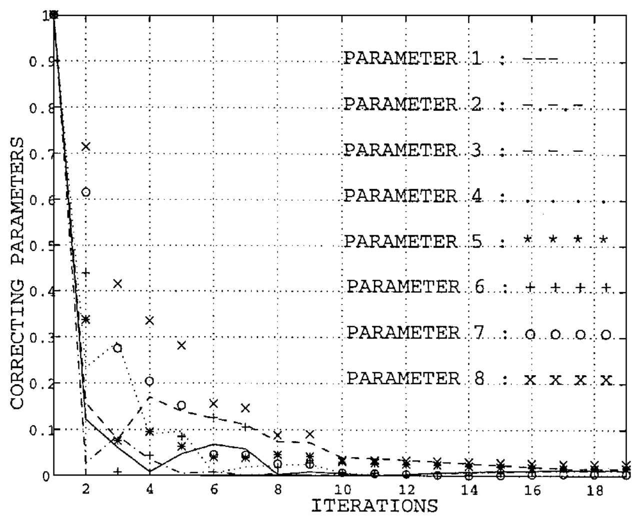


Fig. 3 Correcting parameters.

noise similar to that in the preceding example has been added to the measurements. In this study, the localization step has shown no particular erroneous element. The localization step has been performed by the substructures. In this manner, the truss has been divided into eight parts, as shown in Fig. 2.

Thus, a correcting parameter has been introduced into each damping substructure matrix. Figure 3 displays the convergence of the eight parameters. The initial global error is 0.158, and the final global error is 0.037 after one iteration.

Conclusion

An updating method with an emphasis on damping improvements has been presented. Based on the error on the constitutive relations, the method exhibits foundations that are clearly mechanical in nature. Using a new scaling process that utilizes the dissipated energy, the sensitivity of the error to damping has been greatly improved. Several examples have been presented in which the capabilities of the method to update the damping have been illustrated. Nevertheless, some difficulties such as damping modeling remain, the uniqueness aspect to the updating tasks being affected by the incompleteness of the experimental results and the unavoidable noise inherent in this information.

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